Working With One's Own Tools: Logics and their Meta-Logics

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TABLE OF CONTENTS

INTRODUCTION

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Afterwards I'll examine three liens of response to the argument, and conclude with some remarks on where we're left at the end of things.

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In particular, we're interested in meta-logical results like soundness and completeness.

IDENTIFYING THE META-LOGIC PROBLEM

History of the Meta-Logic Problem

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First, poor citation practices: while this problem has been raised going back decades (e.g. [Meyer 1985]), there are rarely, if ever, citations to people raising the problem in print.

Second, the Meta-Logic problem has been conflated with a number of different nearby problems, which are related but I think conceptually distinct.

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I'll briefly outline each of these and say why I think they are *not* the target of today's talk.

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This quotation is found in discussions of a number of the problems I've listed above, all as a putative representative quote.

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> What is needed, if the two participants to the discussion are to achieve an understanding of each other, is a semantic theory as insensitive as possible to the logic of the metalanguage. ... If both disputants propose semantic theories of this kind, there will be some hope that each can come to understand the other; there is even a possibility that they may find a common basis on which to conduct a discussion of which of them is right. [Dummett 1991: 55]

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To differentiate these problems, let's call the **Problem of Neutral Meta-Language** the problem of determining what logic we ought to use to when settling disagreements between different philosophies of logic.

Williamson on the Neutral Meta-Languages

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> How can this anarchy of different systems be reconciled with the apparently scientific, unphilosophical nature of logic? The answer lies in the role of metalogic. All these systems are normally studied from within a first-order non-modal metalanguage, using classical reasoning and set theory. Scientific order is restored at the meta-level. Not only are the systems susceptible to normal methods of mathematical inquiry with respect to their syntax and proof theory, their model theory is also carried out within classical first-order set theory. [Williamson 2014: 214]

Speaking to the Gentiles

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> So, at least so far as my own involvement in the relevant semantical project was concerned, the *aim* was to "preach to the Gentiles in their own tongue". Granted, having provided the Gentiles with the truth-functional explication of relevant logics that they claimed to seek, most of them have decided, as Routley observes, that the sort of explication proferred was not what they had in mind. Still, Routley and I are entitled to feel a bit miffed about having been taken to task for preaching in Classicalese to the Classicalists. For it is they, and not we, who hold the view that *only* Classicalese makes sense; and that, for anything else to make sense, it must have a Classicalese translation. [Meyer 1985: 1]

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Philosophers of logic who argue in favor of semantically closed views can claim that their meta-logic matches their logic insofar as they collapse the distinction between the two.

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> A natural language (or a formal language that models that aspect of its behaviour) can give its own semantics. Naturally, we can still consider that part of a theory which concerns its own semantic notions (and we might call this the metatheory, though in virtue of the misleading overtones, it would be better to avoid this name altogether), but this will now be a *subtheory* of the main theory. Once we rid ourselves of the misleading notion of a metalanguage, the claim that our own semantic discourse should be consistent has no plausibility. Indeed, semantics is a paradigm example of an inconsistent area. [Priest 2008: 70]

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> A prominent defender of such a view, Timothy Williamson, has recently put his logical-abductivist convictions in action by advancing an argument—henceforth referred to as Williamsons argument or, for short, **WA**—to the effect that the way mathematics is used in the natural and social sciences provides abductive support for classical logic over some of its non-classical competitors. [Horvat & Toader 2025: 579]

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If it does, then the non-classical logic should have to rebuild mathematics according to that logic.

Either way, abductive justification speaks in favor of simply sticking with classical logic, because classical mathematics is so prevalent and theoretically simpler than its alternatives.

Abductive Justification and Meta-Logic

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Additionally, the Meta-Logic Problem does not traditionally involve appeals (either implicitly or explicitly) to abductive considerations.

The Problem of Recovering Mathematics

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Lucas Rosenblatt puts it nicely:

> ...the non-classical theorist can no longer take for granted that standard scientific results are available to her. Some of these results may depend on classical principles that are not valid according to the logical theory she endorses. The case of mathematics is particularly telling. In proving theorems mathematicians often appeal, explicitly or implicitly, to classical principles that are not valid in a number of non-classical logics. Thus, the worry, in a nutshell, is that her favored logic is too weak to do mathematics. Even worse, it might be added that the logic is too weak to do science in general because many of the natural sciences constantly make use of mathematical principles, so the restriction could also affect disciplines other than (pure) mathematics. [Rosenblatt 2022: 389]

Recovering Mathematics and Meta-Logic

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Recovering Mathematics and Meta-Logic

Again, while mathematics will play an important result in the Meta-Logic Problem, we should be careful not to conflate these problems.

The Meta-Logic Problem applies more generally, in that it can also be raised against the classical logician.

Further, the Meta-Logic Problem is not aimed at empirical adequacy or recapturing some portion of conventional science or mathematics.

The Problem of Ordinary Reasoning

Finally, we should distinguish the Meta-Logic Problem from the **Problem of Ordinary Reasoning**, or what [Weber 2022] calls the Feferman Objection:

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> Perhaps the most basic reason for resisting nonclassical metatheory is something like Feferman's injunction that [nothing like sustained ordinary reasoning can be carried on] in most non-classical logics. There are clear pragmatic reasons to want to fall back on classical logic. [Weber 2022: 87 / Feferman 1984: 95]

Ordinary Reasoning and Meta-Logic

While I agree with Weber that pragmatic reasons are the source of most appeals to classical meta-theories for non-classical logics, I disagree that the Meta-Logic Problem is the same as Feferman's Objection.

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In particular, he is concerned with non-classical logics not being able to capture object-language moves like implication introduction or material identity.

On the other hand, the Meta-Logic Problem is aimed at very explicitly non-ordinary reasoning: reasoning about object-languages/logics.

The Meta-Logic Problem: Examples

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To start, let's consider John Burgess' attack on relevant logics:

> How far can a logician who professes to hold that perfectionism is the correct criterion of valid argument, but who freely accepts and offers standard mathematical proofs, in particular for theorems about perfectionist logic itself, be regarded as *sincere* or *serious* in objecting to classical logic? [Burgess 2005: 740]

The Meta-Logic Problem: Examples

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> Reliance on classical theory seems peculiar, at least initially, if one takes the arguments that motivate these nonclassical logics seriously. Someone who is philosophically committed to the need for a paraconsistent logic, say because of contradictions in naive set theory, seems vulnerable to at least a nasty ad hominem if they appear to use an inconsistency-intolerant logic. The barbed question of *hypocrisy* is asked by Burgess about relevant logic... [Weber 2022: 85]

The Meta-Logic Problem: Examples

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> Second, the classicality constraints sidestep what we might call the *hypocrisy problem*: if you take "logically valid" to obey a logic weaker than classical, then you should not ultimately be satisfied with developing your theory of that logic using inferences that are merely classically valid. Yet, development of the metalogic without full classical resources presents added difficulties. [Field 2017: 9]

The Meta-Logic Problem: Examples

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> One insightful way of depicting the classical logician's twofold challenge is in the form of a dilemma. Either the non-classical theorist uses classical logic in the meta-language or she does not. If she does, then that is a serious embarrassment because she is overtly availing herself of principles that are only valid in a logic to which she is objecting. If she does not, then that is also a serious embarrassment, although the source of the embarrassment is now different. Assuming that the principles she actually employs in the meta-theory do not go beyond the resources of the logic that she endorses, the issue is that she must acknowledge that many standard results accepted by the mathematical community have no proof. Is there a way out of the dilemma? [Rosenblatt 2022: 389-390]

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> When a related *provisional-on-local-consistency* answer is tried out for the earlier question as to when TDS can be applied, there *is* a serious problem of overall theoretical coherence, since then a principle whose qualified application at the object level is being justified is assumed in the metatheory. It looks uncomfortably like a case of double standards; one, relevant, standard at the object level and another, classical, standard in the metatheory, when a coherent life should be one or the other all the way (e.g., up the levels hierarchy). [Routley 1984: 174–175]

The Meta-Logic Problem: Examples

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> The right response is to insist on doing ones semantics in the logic in which one believes. If Beall and Restall insist on doing semantics classically, then they are classical logicians for whom non-classical "logics are, if not just an intellectual amusement, then an exercise in applying logic to some more particular activity... But what a strange approach to take, if one believes relevance logic is the correct logic. Why use an alien logic for ones metatheoryand if one does, why trust the result? [Read 2006: 207-208]

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> As a rough starting point, well take logical akrasia to consist of a mismatch between the deductive strength of the background logic one uses to prove metatheoretic results and the logical theory one believes (officially), i.e., a form of incoherence in logical theorizing akin to what we saw in the case of epistemic akrasia. So, in other words, logical akrasia will occur when one explicitly appeals to (or at least implicitly commits to) a logical principle which is not endorsed by ones own theory. [Andersen 2024: 4]

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> And this gets to the most basic pointthat producing some simple objects like truth tables is a reasonable adequacy criterion to which to hold a proposed logico-mathematical program. Whether a system can stand independently is a basic test of its viability. Tools that cannot even build ordinary tables are of little value. Regardless of the other philosophical costs and benefits of such systems, and regardless of ones own predispositions about which ones are preferable to use, it is good to know what the serious options really are. [Weber et al. 2015: 535]

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As noted earlier, in identifying the Meta-Logic Problem we want it to be fairly general, applying not to particular theories or positions but to any suitably developed philosophy of logic.

While all of our examples (with the exception of Andersen) identify it as a problem for non-classical logicians, we can imagine the problem arising for classical logicians as well: what principles ought they (not) appeal to in their meta-logic?

Identifying the Meta-Logic Problem

Similarly, while most of our examples are centered on disagreements between classical logicians and non-classical logicians, we can imagine it arising for a lone non-classical logician wondering what they should accept by their own lights, independently of engagement with other, rival logicians.

Identifying the Meta-Logic Problem

Similarly, while most of our examples are centered on disagreements between classical logicians and non-classical logicians, we can imagine it arising for a lone non-classical logician wondering what they should accept by their own lights, independently of engagement with other, rival logicians.

In this way it is more akin to a traditional problem in mathematics regarding whether we should accept some mathematical result which rests on a controversial principle, axiom or assumption.

The Meta-Logic Problem (Initially) Identified

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The Meta-Logic Problem (Initially) Identified

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Unfortunately this formulation of the problem is open to two initial objections, to which we'll now turn.

OBJECTION 1: META-LOGIC ISN'T FORMAL

The Informality of Meta-Logic

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Meyer, in complaining about the Meta-Logic Problem, gives this objection:

> I have also tried out the following line of argument, again without finding anybody who admitted being persuaded by it. When we reason about logic, we reason *informally*, in English. In particular, our arguments that R is semantically complete were informal arguments, though they were conducted with that degree of precision and conformity to prevailing standards of logical and mathematical rigour as one might expect in work directed toward a logically mature audience. [Meyer 1985: 1]

The Potential Formality of Meta-Logic

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In fact, we have some limited results here using automated theorem provers based (primarily) in intuitionistic logic (e.g. Coq).

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> In principle, such arguments are *formalizable*, in any formal logic
> which is suitably responsive to the canons of *informal* rigour.
> Certainly it is not to be assumed, given an informally valid argu-
> ment, that, were it to be formalized, it is intended that it should
> be formalized *classically*. For that would beg the principal ques-
> tion that one takes to be at issue between the classicist and the
> relevantist: Namely, are informally valid arguments best formal-
> ized according to classical logic, or in a relevant logic? Moreover,
> it would beg the question in favour of the classicist–a point hardly
> to be given up without a fight. [Meyer 1985: 1-2]

Meta-Logic and Formality

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The traditional conception of logic holds that logic is universal, in the sense that it applies to all deductive arguments regardless of their content.

To claim that meta-logic raises no problems because its arguments are not formalizable would be to give up on this traditional conception (and not even in the way that other programs give up other aspects, e.g. relativism).

OBJECTION 2: META-LOGIC ISN'T LOGIC

Meta-Logic, Logic and Mathematics

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That is: they claim that that meta-logic isn't really *logic*, it's *mathematics*.

Thus, the Meta-Logic Problem is misplaced: everyone, including the non-classical logician, can help them to classical meta-logic, without sacrificing their logical principles, because they can help themselves to classical mathematics.

Meta-Logic, Logic and Mathematics

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> The first important point to note is that on most views of logic
> prevalent today, there will be inferences employed in metatheoret-
> ical reasoning which are not logically valid, such as the essentially
> mathematical inferences concerning numbers or sets. The reason
> for this is that metatheory (in its various guises) comprises a col-
> lection of branches of research in *classical mathematics*, including
> set theory, model theory, proof theory, computability theory, and
> with significant connections to topology, universal algebra, cate-
> gory theory, and other areas of contemporary mathematics.

Meta-Logic, Logic and Mathematics

It is very widely agreed by mathematicians that classical logic is the correct logic for mathematics. But the classical mathematics used in various metatheoretical studies routinely outrun what we expect of the canons of logically valid inference. For example, almost all completeness proofs make use of the axiom of choice, in its guise as Zorns lemma. Unless you are a logicist, or are a proponent of an implausibly strong second order version of classical logic, then you should not expect the inference principles applicable in mathematics to be reducible to inference principles which are logically valid.

Meta-Logic, Logic and Mathematics

All this is to say that there are inference principles used in metatheory which even classical logicians are unlikely to countenance as logically valid. Indeed, it seems quite reasonable, absent commitment to logicism, to expect most logicians to admit that there are *extralogical* inference principles needed. That is to say, principles which we might not claim to be logically valid, but which are permissible within the context (of classical mathematics) in which they are used. One may well wonder whether the axiom of choice is justifiable on *purely* logical grounds, but there is little real dispute that it is justifiable at least within the context in which it is commonly used. While it may not be a logical truth, it is certainly (setting aside constructivist objections) a truth of set theory. [Tedder forthcoming: 2]

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▶ Brouwerian intuitionists, who believe that logic is reducible to mathematics

Meta-Logic, Logic and Mathematics Revisited

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First: the claim that classical logic is the correct logic for mathematics.

Second: the claim that these extralogical principles are freely justified absent very special philosophical commitments.

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But I see no intrinsic reason that the mathematical community is the relevant body of experts here, and when we move to philosophers of math the issue is comparatively much more contentious.

Meta-Logic, Logic and Mathematics Revisited

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- ▶ (Most) dialetheists, who believe that dialetheia exist not only in the context of the truth predicate but also concepts like SET

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At least not absent further argument. We now turn to potential avenues they might take to justify using a classical meta-logic in a way consistent with their philosophical principles.

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First, our meta-logic will not generally match our logic insofar as the
meta-logic occurs in an informal/semi-formal setting.

Even when formalizing our meta-logical arguments strictly the arguments
will involve appeals to principles which are not strictly *logically* justified,
because they appeal to mathematical theories.

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yes; to answer otherwise would be to live an incoherent logical life as
Sylvan put it.

But things are not so bleak, because the non-classical logician may still
have routes available to them which are coherent with their overall
philosophy of logic. Lets consider three possibilities.

ROUTE 1: RELATIVISM/PLURALISM TO THE RESCUE

Different Logics for Different Purposes

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But recently views which are designed precisely to deny this intuition have become popular.

These are forms of **logical relativism**, the view that different logics apply to different things (e.g. topics or domains).

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According to these views, the notion of a logic being "correct" is relative
to a domain of discourse.

These views deny the traditional view that logic universal or
topic-neutral, but still maintain that there are logics which are
correct/incorrect for describing or modeling domains of discourse.

The Relativist Answer to the Meta-Logic Problem

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It's not entirely clear what would motivate this position, especially since domain individuation is generally less fine-grained than that, but it is at least a potential route to explore.

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An alternate route that can avoid giving up on this core principle is to be a **global pluralist** about logic and maintain that there are multiple, equally correct logics which apply everywhere.

If there are multiple equally correct logics and one of them is classical logic, than the global pluralist who is also a non-classical logician can help themselves to classical logic in doing meta-logic (as it is applicable everywhere).

Pluralism's Problem: Collapse

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Here lies the various forms of the so-called **Collapse Problems** for (global) logical pluralism, but we can set them aside for this talk.

Relativism, Pluralism and Meta-Logic

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The relativist will have to explain why the logic of meta-logical investigations is what it is, and the global pluralist will have to answer general questions about the coherency of their view.

But it remains open to both to offer a view according to which there need not be a match between the object-language logic (in question) and the meta-logic, without sacrificing their underlying philosophical motivations.

ROUTE 2: RECAPTURE TO THE RESCUE

Classical Recapture

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On the other hand, one primary feature of non-classical logics is that they are **subclassical**, i.e. they have fewer valid arguments than classical logic.

This is the sense in which non-classical logics are weaker than classical logic, and thus less able to prove the meta-logical results we desire (e.g. soundness or completeness).

Technical Recapture vs. Philosophical Recapture

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[Cook forthcoming] provides a general recipe for recapture for a large
class of logics, building on work from [Beall 2013].

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But Dummett is happy to admit that intuitionistic logic can fully mimic classical results in finite, decidable domains (see {Dummett 1975}).

But this does not come for free. We must win our way through to recapturing classical behavior by showing that a domain is decidable, or giving an a priori argument that it is determinate.

The Determinacy of Meta-Logic

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Further, given that Dummett (even the late Dummett who does accept a form of logical pluralism) never gave such an argument despite being fully aware of results regarding the poverty of intuitionistic meta-logic, we should not assume such an argument will be forthcoming.

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Unlike for the intuitionistic or paracomplete logician who merely needs to add in determinacy principles, the paraconsistent logician cannot seemingly stipulate their way into consistency.

As [Anderson & Belnap 1975] argue, no amount of adding premises can guarantee that one is in a consistent situation when those premises can themselves be inconsistent.

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A natural language (or a formal language that models that aspect of its behaviour) can give its own semantics. Naturally, we can still consider that part of a theory which concerns its own semantic notions (and we might call this the metatheory, though in virtue of the misleading overtones, it would be better to avoid this name altogether), but this will now be a *subtheory* of the main theory. Once we rid ourselves of the misleading notion of a metalanguage, the claim that our own semantic discourse should be consistent has no plausibility. Indeed, semantics is a paradigm example of an inconsistent area. [Priest 2008: 70]

The Prospects for Recapture

While it is certainly possible that non-classical logicians can offer a
philosophically substantive argument for classical recapture in meta-logic
(or the relevant bit of mathematics needed for meta-logic), it's certainly
not a given.

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While it is certainly possible that non-classical logicians can offer a philosophically substantive argument for classical recapture in meta-logic (or the relevant bit of mathematics needed for meta-logic), it's certainly not a given.

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For paracomplete logics (e.g. intuitionistic logic), this will generally happen in the form of offering an argument that meta-logic is determinate.

For paraconsistent logics, this will happen via arguing that meta-logic is consistent, and free of the potential of inconsistency.

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> That seems to me a satisfying conclusion: classical logicians reason as if they were God; they are therefore guilty of overweening presumption. [Dummett 2006: ix]

Assuming that meta-logic behaves classical logic is to be guilty of this same overweening presumption, and the non-classical logician ought to be better than that.

ROUTE 3: HYPOTHETICAL META-LOGICAL
RESULTS TO THE RESCUE

Learning from Mathematics

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The Holy Grail of pre-Gdelian mathematics was providing a foundation for mathematics which proved its own soundness and completeness.

After the incompleteness theorems were properly understood, people did not stop attempting to understand mathematics or logic, despite knowing that their goal was impossible.

Instead, they sought to understand more about their theories using relative consistency proofs of various theories to one another.

Hypothetical Meta-Logical Results

We don't have an exact analogue to this in philosophy of logic, because again, the philosophical positions which move people towards these various logics often cast doubts on the theories that we might expect results from (e.g. classical logic/mathematics).

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However, many of these programs are at least open to moving up or down in relative strength (e.g. intuitionistic/intermediate logics and relevant logics).

If that's the case, we can prove what we might call **hypothetical meta-logical results**, i.e. meta-logical results from a theory slightly stronger but within philosophical reach.

Intuitionistic Meta-Logical Results

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For example, it's well known that intuitionistic predicate logic (IPC) is incomplete relative to its own meta-logic and relative complete to a classical meta-logic.

Less well known is that systems slightly stronger than intuitionistic logic are provably complete.

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implies completeness of IPC.

This has led many people (e.g. [Dummett 2000]) to argue that
intuitionists must settle for incompleteness, because MP is
intuitionistically unacceptable.

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But not everyone agrees. For example, another (non-Dummettian) intuitionist, David McCarty, analyzes the situation in the following way:

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Ought we to conclude from the theorems of Gdel and Kreisel that a purely intuitionistic proof of the completeness of [IPC] will always lay beyond our grasp? One should answer "Yes" if there are convincing arguments that MP is not, strictly speaking, intuitionistically correct. Yet these arguments–at least such as are in popular circulation–are not wholly convincing. [McCarty 1994: 103-104]

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> ...MP is no principle of intuitionistic *logic*. What remains highly contestable is [Dummett's] implicit claim–that seems to be required if we are to conclude from the above that MP is constructively inappropriate–that if MP be true at all, it must be true as a matter of logic (given, perhaps, some semantical reflections). One might agree with Dummett over the status of MP in logic (plus, perhaps, semantics) and continue to maintain that MP is a mathematically correct statement...[McCarty 1994: 105]

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First, so long as super-intuitionistic principles like MP are intelligible to the intuitionist, they can note the hypothetical meta-logical result:

If MP is true, then IPC is complete.

Second, intuitionists may be able to engage in some form of recapture when considering these meta-logical results by knowing exactly how much determinacy they need to argue for to get the results they want.

Hypothetical Meta-Logical Results for Other Logics

Things are less clear elsewhere, but to take one example: [Meyer 1985] provides a completeness proof of the relevant logic R in its own meta-logic.

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R is a relatively strong relevant system, but again, so long as it is intelligible to other relevant subsystems of R there are some potential lessons to be learned, even if only on a hypothetical basis.

WORKING WITH ONE'S OWN TOOLS

Planning for the Worst

With these routes discussed, let's now plan for the worst and ask: where are we left if we cannot answer the Meta-Logic Problem and must work with our own tools?

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With these routes discussed, let's now plan for the worst and ask: where are we left if we cannot answer the Meta-Logic Problem and must work with our own tools?

It's important to note that, as [Weber 2022] pointed out, many appeals to classical meta-logics are simply pragmatic in nature.

Some people take it even further, arguing that the process is a waste of time, even while noting it can be done.

Meyer on the Chore of Being Principled

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> Where the classicist has the edge, with respect to his thesis that valid arguments may be rigourously formalized in a classical theory, is that he has a lot more practice. By contrast, only in a few areas do we know what fully formalized relevant theories look like. Important technical and even philosophical problems must be overcome. To be sure, one of the reasons that the classicist has the edge is that he *oversimplifies*, trusting that distinctions that we have always wished to make (between co-extensional properties, for example) are of no ultimate import, Since nobody has shown that rational thought can do without the waived distinctions, this leaves a big mess to be cleaned up by workers in any program of formal reconstruction of informal arguments and theories.

Meyer on the Chore of Being Principled

But, where a host of problems continue to await workers in relevant projects, in particular, surely one of the least important of all is the relevant formulation of *metalogic*. Relevant logics may have found their Booles (and be sure to voice the consonant, dear reader, even if some of their arguments make you wince). But, since they have not yet found their Whiteheads and Russells, it is a little early to ask for a Carnap. [Meyer 1985: 2-3]

Some Room for Optimism

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As he puts it shortly after, **Meyer's Thesis** is that all good arguments inevitably have relevantly valid formal counterparts (if that's not optimistic, I have no idea what is!).

Nonetheless, there is ultimately *some* room for optimism here, and we can detail some results.

Working With One's Own Tools: Relevant Logics

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Other relevant logicians (e.g. [Read 1988]) are happy to take this result
as given.

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It's likely that some intermediate logics are provably complete, although less work has been done on them and especially in their own meta-logics.

Who's Afraid of Incompleteness?

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> just as Gdel's incompleteness results did not destroy the interest in investigating proof-theoretical questions relating to classical theories, so the fact that we never expect to have a complete formalization of any intuitionistic theory should not deter us from studying similar questions in this area. [Dummett 2000: 211]

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> The standard practice of logicians, in treating of any well-defined fragment of logical theory, is to seek to define two parallel notions of logical consequence, one syntactic and the other semantic, and then attempt to establish a relation between them. The ideal is to establish their extensional equivalence. [Dummett 1978: 290]

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But what underlies this practice, other than mere technical interest or convenience?

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> Kneale, for instance, urges that only complete systems be allowed within the scope of logic. ... Kneale argues like this: the fact that a theory is incomplete shows that its basic concepts cannot be fully formalised, and this, in view of the essentially formal character of logic, justifies excluding such theories from its scope. So, interestingly, Kneale is proposing completeness as the test of a system's being 'purely formal'; he connects the precise idea of completeness with the vaguer notion of topic-neutrality. [Haack 1978: 6-7]

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But in his foundational defense of second-order logics Stewart Shapiro argued that the epistemic gains of completeness proofs are overrated, and that it is unlikely to be a strong criteria of adequacy of logic. (See [Shapiro 1991] and [Shapiro 1999].)

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> Unless one demands fundamental changes in the interpretations of the connective symbols, as in [Veldman 1976], intuitionistic formal logic remains incomplete. For this, intuitionists are generally thankful: the incompleteness of formal logic is further confirmation of their insistence [that] logic must be ancillary to mathematics. [McCarty 2005: 373]

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He offers at least four arguments against the idea of incompleteness in an earlier paper, [McCarty 1991], but all of these are unlikely to be intelligible let alone convincing to those without strong constructivist intuitions.

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But even when noting this, Dummett restricts himself to emphasizing soundness:

> What a semantics for a logical theory has to be able to show is, first, that the rules of inference we ordinarily employ are in fact valid, that is, that they are justified in the sense that that truth is preserved as we pass from premises to conclusion. [Dummett 1978: 311].

CONCLUSION

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These concerns often (but not always) permeate beyond just logic to other areas of inquiry, including mathematics, and thus meta-logic.

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One option is to give up on the universality of logic; sacrificing one principle for another.

Another is to engage in classical recapture, although these programs can be philosophically tough to justify.

Another response is to state hypothetical results that do not hold for your system but hold in other, stronger systems, as we do in arithmetic.

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Sure, it's hard work, but nothing good comes easy.

If we want to be principled, we should prefer honest toil over theft.

Especially so when what's available to steal is rotten anyway.

Thanks!

Now time to point out where you think *I* was guilt of reasoning as if I were God.

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